INDO SWISS JOINT RESEARCH PROGRAMME (ISJRP)

RESEARCH FELLOWSHIPS

EXCHANGE GRANT REPORT

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Part 1 - General Information

Project Title: Effect of stochastic perturbation on allelopathic phytoplankton model
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Part 2 - Exchange Participant(s) Details

VISITING SCIENTIST

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Part 3 - Scientific & Technical Information
3.1 Purpose of visit

Study the effect of stochasticity on a model of phytoplankton allelopathy. In this project we first analyzed the stability of the non stochastic model systems. Next we consider the model with stochastic term and studied the stability.

3.2 Short description of work carried out during the visit

The term plankton refers to the freely floating and weakly swimming organisms within aquatic environment. The plant species commonly known as phytoplankton are unicellular and microscopic in size. Phytoplankton has a significant utility in marine life and they play a vital role at the base of the marine food chain. They also control the global carbon cycle which has a significant impact on the climate regulation. The regular change and abrupt fluctuation of phytoplankton density within aquatic environment are controlled by several factors, e.g., variation of available nutrients, environmental forcing due to seasonal change in environment and many others. The first mathematical model for allelopathic interaction between two competing species is introduced by Maynard-Smith. The model is based upon a two-species Lotka-Volterra competition model with an additional term to take into account the effect of toxic substances released by one species to another.

The classical Lotka-Volterra model for two competing phytoplankton species is governed by the following system of nonlinear ordinary differential equations

\[
\frac{du(t)}{dt} = u(t)(k-a \cdot u(t) - b \cdot v(t)) \\
\frac{dv(t)}{dt} = v(t)(l-c \cdot v(t) - d \cdot u(t)),
\]

where \( u(t) \) and \( v(t) \) denote the densities of two phytoplankton species. \( k, l > 0 \) are the cell proliferation rates per time unit, \( a, c > 0 \) are the rates of intra-specific competition for the first and the second species respectively, and \( b, d > 0 \) stand for the rates of interspecific competition between the first and the second and between the second and the first species respectively.

Next we assume that the first species is capable to releases allelopathic substances which stimulate the growth of the second species. As we have discussed in the introduction, we incorporate an additional term into the growth equation of the second species to model the allelopathic interaction as follows

\[
\frac{du(t)}{dt} = u(t)(k-a \cdot u(t) - b \cdot v(t)) \\
\frac{dv(t)}{dt} = v(t)(l-c \cdot v(t) - d \cdot u(t) + r \cdot u(t)v(t)),
\]

where \( r \) denotes the rate of allelopathic substance released by the first phytoplankton species. We show the existence uniqueness and stability of the above model system.

Next we extended the deterministic model system to a stochastic delay differential equation by incorporating multiplicative white noise terms in the growth equations of both species. We studied the mean square stability of the model using a suitable Lyapunov functional. To illustrate the theoretical results, we performed some numerical simulations.

3.3 Outcomes
We have extended the model system by introducing multiplicative noise term where strength of the noise is proportional to distance of $u(t)$ and $v(t)$ from their equilibrium levels. We have obtained the stability conditions etc. for the stochastic delay differential equations (SDDE). It is noticed that stability of SDDE system demands some additional restrictions. In some cases the magnitude of discrete time delay plays a crucial role to determine the point, where the solution trajectory will approach the equilibrium level or not in the sense of probability.

3.4 Future collaboration with host institution

In future we will work on some area of mutual interest. I really enjoyed working there.

3.5 Various comments

Everything was very well. I would like to thanks a lot EPFL-DST for funding and Prof. Norbert Hungerbuehler for giving me such a wonderful opportunity to visit his department. Fribourg was a wonderful place and people were very friendly.

3.6 Projected publications/articles resulting or to result from the exchange

Syed Abbas, Malay Banerjee, Norbert Hungerbuehler, Existence, uniqueness and stability analysis of allelopathic stimulatory phytoplankton models, Communicated.